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RESEARCH IN APPLIED MATHEMATICS RELATED TO MATHEMATICAL SYSTEM --ETC(U)
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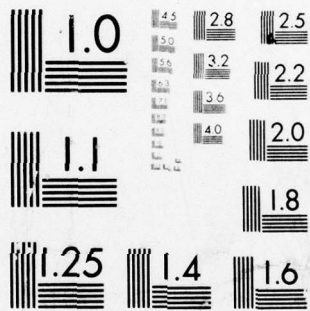
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University of Florida, Gainesville, FL 3261110. PROGRAM ELEMENT, PROJECT, TASK
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Patricia B. Rambo
University of Florida, Gainesville, FL 32611

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18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official
Department of the Army position, unless so designated by other authorized
documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

mathematical system theory applied algebraic geometry
linear systems
linear systems over a ring
nonlinear realization theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report deals with research results obtained at the Center for
Mathematical System Theory, University of Florida, Gainesville, Florida 32611
in the field of mathematical system theory. Special emphasis was given to
the following areas:(1) Linear system theory over a field: parametrization of multi-input,
multi-output systems and the geometric structure of classes of systems of con-
stant dimension. — next page

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20. ABSTRACT (Cont.)

→(2) Linear systems over a ring: development of the theory for very general classes of rings.

(3) Nonlinear system theory: basic properties of polynomial (discrete-time) systems and their realization theory.

The present report is only a brief summary of findings, which are far too technical to be explained in a few pages. The results are being gradually published in the open literature. For details, refer to previous proposals and/or to the publications listed in the references.

The program is expected to continue, the bulk of effort now being devoted to nonlinear problems.

THE FINDINGS OF THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF ARMY POSITION, UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

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RESEARCH IN APPLIED MATHEMATICS RELATED TO
MATHEMATICAL SYSTEM THEORY

Final Technical Report

June 1977

U. S. Army Research Office

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DAAG29 76 G 0203

1 May 1975 through 18 April 1977

CENTER FOR MATHEMATICAL SYSTEM THEORY
UNIVERSITY OF FLORIDA
GAINESVILLE, FL 32611

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ABSTRACT

This report deals with research results obtained at the Center for Mathematical System Theory, University of Florida, Gainesville, FL 32611 in the field of mathematical system theory. Special emphasis was given to the following areas:

- (1) Linear system theory over a field: parametrization of multi-input, multi-output systems and the geometric structure of classes of systems of constant dimension.
- (2) Linear systems over a ring: development of the theory for very general classes of rings.
- (3) Nonlinear system theory: basic properties of polynomial (discrete-time) systems and their realization theory.

The present report is only a brief summary of findings, which are far too technical to be explained in a few pages. The results are being gradually published in the open literature. For details, refer to previous proposals and/or to the publications listed in the references.

The program is expected to continue, the bulk of effort now being devoted to nonlinear problems.

THE FINDINGS OF THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF ARMY POSITION, UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

1. SUMMARY OF ACTIVITIES UNDER GRANT

a. Technical work.

The work supported under this grant is on a very advanced mathematical level, requiring extensive technical background on the part of the reader. The present report is intended only as a guide for those interested in getting acquainted in depth with the results obtained.

For a relatively elementary introduction, the following two sources are suggested

(i) The review sections of the research proposals in 1974, 1975, and 1976.

(ii) Chapter 1 of Sontag's dissertation (see SONTAG [1976d]).

Below are some historical comments about the evolution of the research, with precise references to already published results.

Algebraic theory of linear systems. The principal effort here was to describe classes of systems of constant dimension from the point of view of algebraic geometry and to relate the results obtained to questions involving the transfer-function description of linear systems. In regard to the first question, early ideas of the Principal Investigator are summarized and extended in HAZEWINKELE and KALMAN [1976]; this paper is followed by more detailed exposition by HAZEWINKELE himself (Erasmus University of Rotterdam report, being published). The second question involves a large amount of unpublished work by KALMAN which is now evolving into a research monograph (see also lecture notes at Royal Institute of Technology, Stockholm (1975) as well as at Swiss Federal Institute of Technology, Zürich (1975-77)). Closely related to this is the work of HAUTUS and HEYMANN [1976].

Linear systems over rings. The earlier work of ROUCHALEAU and ROUCHALEAU and WYMAN has been vigorously continued and extended by SONTAG [1975, 1976a-c]. The last paper is a detailed and tutorial introduction to work in this area, at the Center and elsewhere, until about 1975. A certain completion of the theory was recently accomplished by ROUCHALEAU and SONTAG [1977].

Algebraic theory of nonlinear systems. The main effort here was the dissertation of SONTAG [1976d], some of the simpler results of which have already been published in SONTAG and ROUCHALEAU [1976]. The main results of SONTAG concern the definition of a slightly restricted class of polynomial (discrete-time) systems for which an elegant and algebraically effective realization theory can be developed. This work opens up many new problems which allow the application of advanced tools from algebraic geometry.

Independently of this work, the Principal Investigator has solved a problem open since 1968 by giving a canonical realization for bilinear response functions. See KALMAN [1977]. This result, coupled with the proof of the uniqueness theorem for canonical realizations (see SONTAG [1967d]) is probably the first time that system-theoretic methods have provided a complete insight into a class of nonlinear problems. (The well known theory of the so-called internally bilinear systems, in the sense of BROCKETT, ISIDORI, FLIESS, etc., has not yet reached this stage.)

b. Personnel supported under the grant.

Prof. R. E. Kalman (Principal Investigator)

Dr. E. Sontag (pre and then postdoctoral student)

Mr. T. Matsuo, Y. Yamamoto, F. Hamano (predoctoral fellows)

c. Visitors at the Center for Mathematical System Theory (in part supported from grant funds)

1975/76 Mr. A. Antoulas, ETH Zürich (6 weeks)

Prof. M. L. J. Hautus, Eindhoven Technological University (6 months)

Prof. M. Hazewinkel, Erasmus University, Rotterdam (2 months)

Prof. M. Heymann, Technion, Israel (11 months, sabbatical leave)

Prof. S. K. Mitter, MIT (2 weeks)

Dr. Y. Rouchaleau, Ecole des Mines, Paris (6 weeks)

Dr. G. Sonnevend, Hungarian Academy of Sciences (10 months)

1975/76

Shorter Visits

Prof. D. H. Jacobson (University of the Witwatersrand)

Dr. S. I. Marcus (University of Texas)

Dr. B. P. Molinari (Australian National University)

Dr. T. J. Tarn (Washington University)

Prof. H. Tokumaru (Kyoto University)

1976/77

Prof. R. W. Brockett, Harvard University (sabbatical leave; 5 months)

Dr. M. Fliess, University of Paris (6 weeks)

Dr. V. Kučera, Czechoslovakian Academy of Sciences (4 months)

Dr. E. D. Sontag, (Postdoctoral Fellow, 11 months)

Shorter Visits

Prof. E. Kamen (Georgia Institute of Technology)

Prof. H. Sussmann (Rutgers University)

2. LIST OF PUBLICATIONS

W. DICKS and E. D. SONTAG

[1977] "Sylvester domains", to appear.

M. L. J. HAUTUS and M. HEYMANN

[1976] "Linear feedback - an algebraic approach", to appear.

M. L. J. HAUTUS and M. HEYMANN and R. J. STERN

[1976] "Rest point theorems for autonomous control systems", to appear.

M. HAZEWINKEL and R. E. KALMAN

[1976] "On invariants, canonical forms, and moduli for linear constant, finite-dimensional dynamical systems", in ALGEBRAIC SYSTEM THEORY, Springer Lecture Notes in Economics and Mathematical Systems, No. 131.

R. E. KALMAN

[1977] "Canonical realization of bilinear response functions", to appear in Ricerche di Automatica.

Y. ROUCHALEAU and E. D. SONTAG

[1977] "On the existence of minimal realizations of linear dynamical systems over Noetherian integral domains", to appear in J. Computer and Systems Sciences.

E. D. SONTAG

[1975] "On certain questions of rationality and decidability", J. Comp. Syst. Sci., 11: 375-381.

[1976a] "On linear systems and noncommutative rings", Math. Systems Theory, 2: 327-344.

[1976b] "On finitely accessible and finitely observable rings", J. of Pure and Applied Algebra, 8: 97-104.

[1976c] "Linear systems over commutative rings: a survey", Ricerche di Automatica, 7: 1-34.

[1976d] "On the internal realization of polynomial response maps", doctoral dissertation, University of Florida.

[1977a] "On the observability of polynomial systems", to appear.

[1977b] "On finitary linear systems", to appear.

[1977c] "The lattice of minimal realizations of response maps over rings", to appear.

[1977d] "On first-order equations for multidimensional filters", to appear.

LIST OF PUBLICATIONS (cont.)

E. D. SONTAG and Y. ROUCHALEAU

[1976a] "On discrete-time polynomial systems", J. Nonlinear Analysis, 1: 55-64.